

# **Assimilation of Three-Dimensional Phase-Resolved Wave-Field Data Using An Efficient High-Order Spectral Method**

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## **LONG-TERM GOAL**

The long-term goal is to develop a robust and efficient computational tool for direct phase-resolved large-scale simulations of nonlinear ocean wave-field evolutions under offshore and coastal environments including realistic effects due to nonlinear wave-wave interactions, variable current, wave-breaking dissipation, bottom reflection and refraction, and wind-wave interactions.

## **OBJECTIVES:**

The specific scientific objectives of this program are to:

1. Extend and apply an existing phase-resolved simulation program, a powerful high-order spectral method (HOS) for nonlinear wave-wave interactions, to assimilate realistic ocean wave-field data and to predict long-time evolutions of such nonlinear wave-fields
2. Obtain realizable initial wave-fields for phase-resolved simulations from either of the following: (i) multiple wave probe records; (ii) ocean surface images from, for example, Scanning Radar Altimeters (SRA), Synthetic Aperture Radars (SAR), or Focused Phased Array Imaging Radars (FOPAIR); and (iii) three-dimensional wave spectral specifications
3. Provide a framework for cross-calibration/validation of laboratory and field data and a quantitative assessment of the range of validity and accuracy of phase-averaged wave-prediction models
4. Investigate deterministic mechanisms for wave focusing and localization due to nonlinear wave interactions with variable currents and bottom topography using direct HOS simulations

## **APPROACH**

An efficient high-order spectral method (HOS) for the phase-resolved simulation of nonlinear surface wave dynamics is extended to practical applications. For data assimilation and specification of the initial conditions for direct phase-resolved time simulations, an effective wave reconstruction scheme based on the multi-level iterative optimization (Wu, Liu & Yue 2000, 2001) is applied. HOS is a pseudo-spectral-based method that can account for nonlinear wave interactions to arbitrary high order

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( $M$ ). The method is extremely efficient as it obtains exponential convergence and linear computational effort with respect to the order ( $M$ ) and the number of wave modes ( $N$ ). HOS is an ideal approach for direct simulations of large space-time domain nonlinear evolution of wave-fields. The efficacy of HOS for the study of mechanisms of nonlinear wave dynamics in the presence of atmospheric forcing, long-short waves, finite depth and depth variations and bodies has been well established (e.g. Liu & Yue 1998).

## WORK COMPLETED

The main focuses are on the application of HOS simulations to obtain direct comparisons with the wave-basin measurements of steep wave-field evolutions and the study of fundamental mechanisms of wave focusing/blocking and localization by variable currents and irregular or near-regular bottom topography. The specific works completed include:

- Improvement of efficiency and robustness of HOS and the wave reconstruction scheme for long-time and large-scale nonlinear wave-field simulations, and test/validation of HOS computations on distributed-memory parallel platforms (IBM SP3).
- Application of the wave reconstruction scheme to wave-basin data, and direct comparisons of HOS simulations and wave-basin measurements of steep wave-field evolutions (Wu, Liu & Yue 2001).
- Investigation of the nonlinear mechanisms of wave reflection/refraction, blocking, and focusing by variable ambient currents, and performance of direct HOS simulations of steep wave-field evolutions through variable current fields.
- Investigation of the fundamental mechanisms of wave localization and high-order wave-bottom resonance for wave propagation over long irregular or quasi-regular bottom variations; and study of the effects of two- and three-dimensional wave localization and wave-bottom resonance upon near-shore wave-field evolution (Burr 2001).
- Study of resonant interactions between ambient surface waves and steady Kelvin waves behind fixed objects in currents (or moving ships). It is found that the quartet resonant interactions between Kelvin waves and ambient waves can occur along certain rays in the Kelvin wake. As a result, new propagating waves with significantly large amplitudes can be generated along the resonance rays (Zhu, Liu & Yue 2001a).
- Numerical investigation of three-dimensional instability of nonlinear standing surface waves in a rectangular or circular basin/harbor. Beyond a threshold wave steepness, both plane standing waves or standing waves in a circular basin are found to be unstable to a small three-dimensional disturbance. The root cause for such instability is found to be due to the third-order (quartet) resonant interactions between the base standing wave and small disturbances (Zhu, Liu & Yue 2001b).

## RESULTS

Figure 1 shows the quantitative comparison of the reconstructed wave elevation and horizontal velocity of a steep irregular long-crest wave-field and the wave-basin measurements. The irregular wave-field is given by the Pierson-Moskowitz spectrum with peak period of 0.89s and significant wave height of 0.09m. The wave-basin (46m × 30m) experimental data is provided by Professor M.H. Kim

of the Texas A & M University. To illustrate the importance of nonlinear effects in wave reconstruction, the computational results obtained with fully nonlinear HOS simulations and the linear and second-order theories (Zhang *et al* 1999) are compared in figure1. It is seen that the linear and second-order theories over predict the velocity by about 100% and 20% respectively although they obtain correct predictions of wav elevation. The nonlinear HOS simulations correctly predict both wave elevation and velocity. This result indicates that fully nonlinear effects should be considered in determining wave kinematics based on wave elevation measurements in practice.

Figure 2 shows the wave energy spectra of nonlinear long-crest wave fields in the presence of following and opposite slowly variant currents, obtained using direct long-time HOS simulations (with order  $M=4$  and  $N=O(10^4)$  modes). The initial wave field is constructed from a Pierson-Moskowitz wave spectrum (peak period of 2.4s and significant wave height of 0.29m). Flowing current (with  $U_{max}=0.7\text{m/s}$ ) reduces the wave amplitude while opposite current (with  $U_{max}=-3.0\text{m/s}$ ) increases the wave amplitude. For the opposite current, due to reflection of short waves by the current, the wave spectrum at high frequency is reduced. Sample snapshots of wave profiles of the portion of the wave fields in the absence of current and in the presence of opposite and following currents are compared in figure 3. Clearly, following current is seen to increase wavelength while decrease wave amplitude. In contrast, opposite current shortens wavelength but increases wave amplitude.

## IMPACT/APPLICATION

The present work is a first step toward direct computational prediction of realistic ocean wave-field evolutions without phase-average approximations. It can provide a framework for cross-calibration/validation of laboratory and field data and a quantitative assessment of the range of validity and accuracy of phase-averaged wave-prediction models. It will also be invaluable to improving our understanding and interpretation of remotely-sensed sea surface images.

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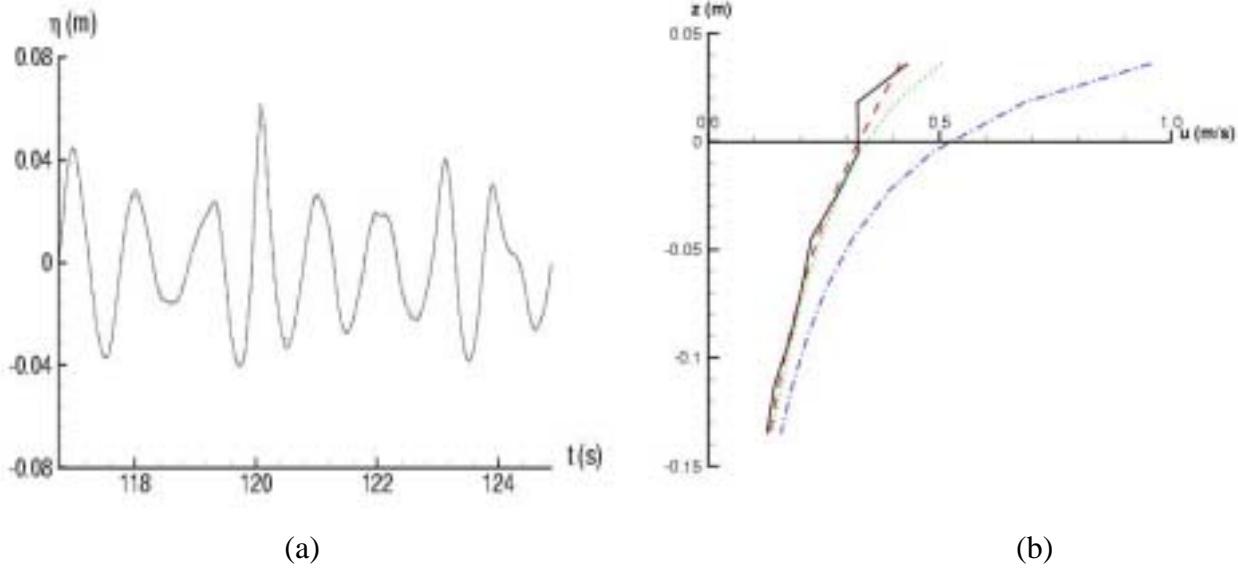
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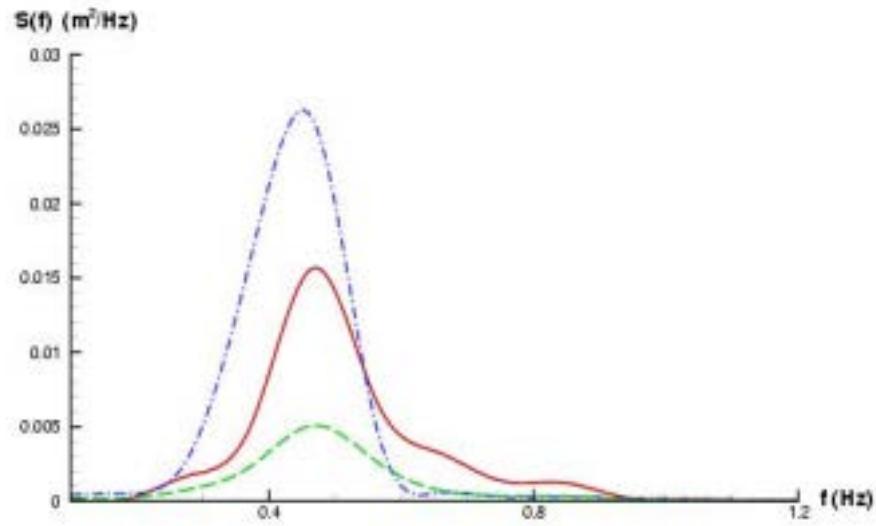
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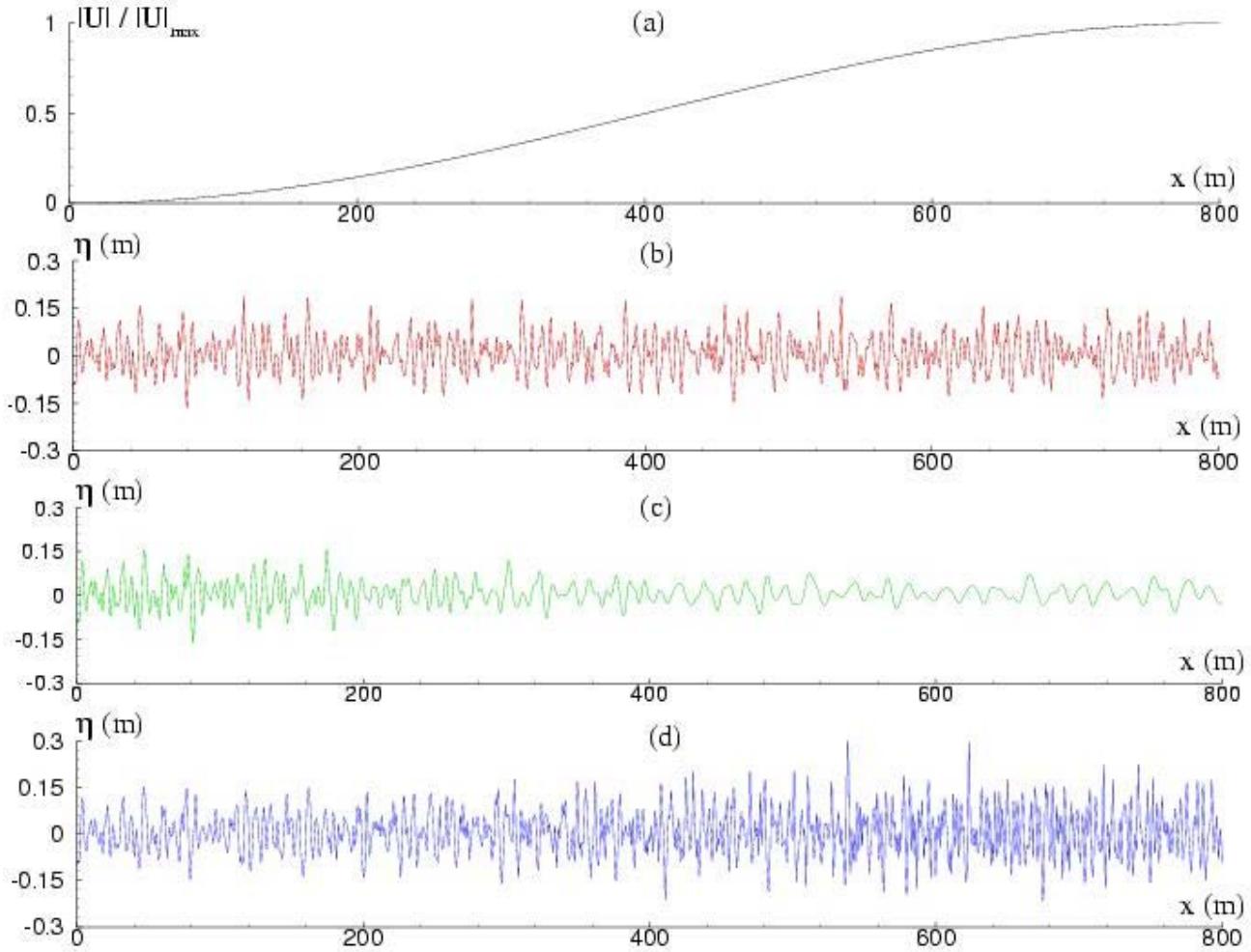
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**Figure 1.** Comparison of computed versus experimentally measured free-surface elevation at a given point (a) and horizontal velocity on a vertical section (b) of a two-dimensional irregular wave-field: experiment (—) and computations with fully nonlinear HOS simulation (---), second-order theory (.....) and linear theory (---.---). (Linear and second-order theories over predict the velocity by about 100% and 20% respectively although they obtain correct wave elevation prediction. Nonlinear theory correctly predicts both wave elevation and velocity. )



**Figure 2.** Comparisons of wave energy spectra of long-crest wave field in the presence of opposite current (---), no current (—), and following current (---), obtained by direct nonlinear HOS simulations. (Wave spectrum is increased by opposite current but decreased by following current.)



**Figure 3.** Snapshots of surface wave profiles of a long-crest irregular wave field in the presence of (a) slowly variant current with maximum speed: (b)  $U_{max}=0$ , (c)  $U_{max}=0.7$  m/s, and (d)  $U_{max}=-3.0$  m/s. (Following current reduces wave amplitude and increases wavelength. In contrast, opposite current increases wave amplitude, shortens wavelength, and reflects waves.)